Water from Low-Permeability Sediments and Land Subsidence

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Abstract. Seepage pressures are part of the neutral or nondeformative stresses acting in a groundwater basin. The reduction of these pressures gives rise to a stress transfer from neutral to effective. The increase in effective stresses is exclusively responsible for measurable deformations of the land surface. The amount of land subsidence or groundwater recovery from compressible confining layers depends upon the specific storage of the strata and the average head change within them. Expressions for the specific storage are obtained from both consolidation theory and conservation principles of compressible flow. Average head changes are identified on a depth-pressure diagram in terms of head changes in adjacent aquifers and are referred to as 'effective-pressure areas.' The geometry of the effective-pressure area is shown to depend upon the thickness of the compressible strata, the magnitude of artesian pressure decline, the manner in which the basin is developed, and time. These factors are embodied in equations that quantitatively describe the release of stored water from compressible confining layers resulting from their vertical compression in areas of land subsidence.

INTRODUCTION

The conservation principles of classical physics upon which most engineering analysis is based typically lead to one or more differential equations describing the performance of the system. Analytical treatment of compressible flow through compressible mediums, for example, is aided by the continuity equation

$$\text{Inflow} - \text{outflow} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] \Delta x \, \Delta y \, \Delta z \qquad (1)$$

which, for negligible changes in horizontal length, can be restated as the time rate of change in fluid mass

$$\frac{\partial(\Delta m)}{\partial t} = \left[\frac{\rho\theta\partial(\Delta z)}{\partial t} + \frac{\rho\Delta z\partial\theta}{\partial t} + \frac{\Delta z\theta\partial\rho}{\partial t}\right] \Delta x \, \Delta y \qquad (2)$$

and, ultimately, the descriptive differential equation [*Jacob*, 1950]

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{\beta\theta\gamma_\omega + \alpha\gamma_\omega}{k}\frac{\partial h}{\partial t} \qquad (3)$$

In (1) inflow minus outflow is the net inward flux in a porous element, ρ is fluid density, v (x, y, z) are velocity components, Δx, Δy, and Δz are dimensions of the element; in (2) $\partial(\Delta m)/\partial t$ is the time rate of change in fluid mass and θ is porosity; and in (3) h is total head, k is the coefficient of permeability, and the quantity $\beta\theta\gamma_\omega + \alpha\gamma_\omega$ is the specific storage (s_s), where β is the compressibility of water, α is the vertical compressibility of aquifer material, and γ_ω is the unit weight of water.

Solutions to the descriptive differential equation stated above have two general applications in hydroscience. In groundwater problems, expressions for the two-dimensional form of (3) allow determination of the volume of water released from storage from a compressible aquifer owing to a decrease in hydraulic pressure caused by pumping [*Theis*, 1935; *Jacob*, 1950]. In soil mechanics problems, expressions are available for an equation similar to (3) but for one-dimensional flow that allow determination of the rate of consolidation due to an increase in hydraulic pressure and subsequent drainage from compressible clays [*Terzaghi*, 1925]. Terzaghi's work implies a depletion factor or change in water content in compressible units with time due to changes in their stress-strain relationships. In this paper, these principles are translated into more familiar hydrologic terminology for study of land subsidence.

SPECIFIC STORAGE OF COMPRESSIBLE CONFINING LAYERS

The derivation of (3) as it pertains to unsteady flow of water in a compressible aquifer is well understood by hydrologists. For the case of unsteady vertical flow in compressible confining layers, it is only necessary to consider Jacob's treatment of the transient change in fluid mass equation (2). According to *Jacob* [1950] the first term of (2) deals with change in vertical direction (Δz) and is therefore related to vertical compressibility (α). The second term deals with porosity (θ) of the element and is also related to vertical compression, since the dimensional change in the x and y directions is assumed to be negligible. The third term refers to change in fluid density and depends upon fluid compressibility (β). Hence, the first and second terms of the transient change in fluid mass equation are functions of effective or intergranular pressure, and both terms express the vertical compression of the element. Further, in a compressible confining layer, the volume of water obtained from expansion of water is negligible compared with that obtained through a change in porosity, so that the third term of (2) can be neglected. The descriptive differential equation is then expressed

$$\partial^2 h/\partial z^2 = (\alpha\gamma_w/k')(\partial h/\partial t) \qquad (4)$$

where k' is vertical permeability.

The head (h) in (4) is an expression for total head, or sum of the static and transient pore-water pressures. In confined aquifer systems, only heads due to transient pore-water pressures cause flow that results in consolidation. By considering the static head invariant with respect to time, (4) can be rewritten

$$\partial^2 u/\partial z^2 = (\alpha\gamma_w/k')(\partial u/\partial t) \qquad (5)$$

where u is the excess in pore-water pressure.

In this equation, $\partial^2 u/\partial z^2$ denotes change in excess pore-water pressure gradient, $\partial u/\partial t$ denotes rate of change of excess pore-water pressure, and $(k'/\alpha\gamma_w)\ \partial^2 u/\partial z^2$ denotes the volume of water expelled from the voids per unit surface area per unit time.

The specific storage is defined as the volume of water that a unit volume of confining layer releases from storage, owing to its compression when the average excess pressure within the unit volume undergoes a unit decline, and is expressed

$$s_s' = \alpha\gamma_w \qquad (6)$$

From (3) and (5), the specific storage of a confining layer is similar to the specific storage of an adjacent aquifer, differing only in that compressibility of water has been neglected. Further, from the descriptive differential equations, the ratio s_s'/k' influences the response of excess pore water in a confining layer in the same manner as the ratio s_s/k influences the response of a groundwater system to a pumping stress. For a compressible aquifer, the smaller this ratio is the shorter is the duration of time required to develop a cone of depression. For a confining layer, the time required for development of a 'cone of depression' due to vertical movement of water out of the layer, more appropriately thought of as time required to achieve full consolidation, also depends upon the ratio s_s'/k'. The larger the compressibility or smaller the permeability, the greater is the time required to re-establish steady-flow conditions in the confining layer.

Stress-strain relationships. The concept of storage in clay beds implies acceptance of a storage factor or coefficient for such units, and solution requires analysis of changes in stress-strain relationships over long periods of pumping. Consequences of these changes are well documented in the literature: progressive land subsidence in the San Joaquin Valley, California [*Poland and Davis*, 1956; *Poland*, 1961]; the upper Gulf coastal region, Texas [*Winslow and Wood*, 1959]; the Savannah area, Georgia [*Davis et al.*, 1963]; and Las Vegas Valley, Nevada [*Domenico et al.*, 1964]. Other prominent localities include Mexico City [*Cuevas*, 1936] and London [*Wilson and Grace*, 1942]. Analytical description of this phenomenon where it is controlled by pore compressibility is best accomplished by examination of the stress-strain relationships in compressible clays.

With expulsion of water from the voids of an elemental clay volume, the volume decreases, resulting in a vertical shortening or decrease in thickness. The change in vertical dimension has been shown to be related to the compressibility of the element. By definition

$$\alpha = 1/E_c \qquad (7)$$

where E_c is the bulk modulus of compression. The bulk modulus of compression can be identified as a stress-strain ratio

$$E_c = (\Delta \text{ stress})/(\Delta \text{ strain}) = [\Delta\bar{\sigma}/(\Delta H/H_0)] \quad (8)$$

where $\Delta\bar{\sigma}$ is change in effective stress and $\Delta H/H_0$ is change in elemental volume for a unit element, or simply the ratio of change in height (ΔH) to original height (H_0).

Confining layers, whether of unconsolidated or indurated rock, follow Hooke's law of deformation for very small loading increments. With reference to the analogy of Hooke's law, (8) can be restated

$$(\Delta H/H_0)\ (\text{strain}) = (1/E_c)\ \Delta\bar{\sigma}\ (\text{stress}) \quad (9)$$

where E_c is a constant of proportionality. As will be demonstrated, measurable strains of a compressible confining layer resulting from an instantaneously applied stress are time dependent.

Figure 1 is a diagrammatic representation of the void ratio of a unit element of confining layer. The dimensions of the element consist of the height of solids (H_s) and the height of voids (H_v). The original void ratio is defined as the quotient of the volume of voids divided by the volume of solids. Since the solids are assumed to be incompressible, changes in elemental height (ΔH_0) are proportional to changes in void ratio (Δe). The relative compression of the element or relative amount of water loss per unit height can be expressed

$$\Delta H/H_0 = \Delta e/(1 + e) \quad (10)$$

The change in void ratio is assumed to be directly proportional to change in effective pressure, or $\Delta e = a_v\ \Delta\bar{\sigma}$ where a_v is the coefficient of compressibility, or rate of change of void ratio with respect to rate of change of effective pressure causing consolidation. In soil mechanics usage, a_v is the slope of the line obtained by plotting void ratio versus pressure for test specimens [*Terzaghi and Peck*, 1948]. As a_v is inherently variable for any given diagram, its practical use is limited. However, it is a useful soil characteristic and can be considered constant for small changes in pressure.

Substituting $a_v\ \Delta\bar{\sigma}$ for Δe in (10) and placing the new expression in (9) results in

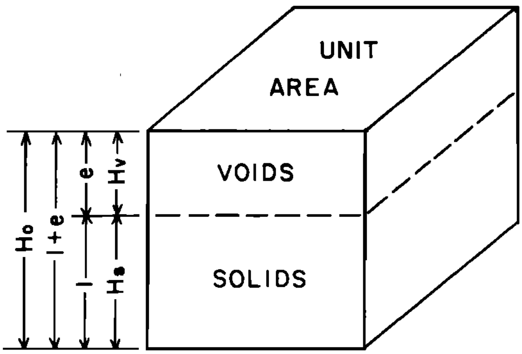

Fig. 1. Schematic representation of void ratio for an element of confining layer.

$$1/E_c = a_v/(1 + e) = \Delta e/[\Delta\bar{\sigma}(1 + e)] \quad (11)$$

For a one-dimensional system, the quantities expressed in (11) represent the height of a pore-water column expelled from a unit element when the effective pressure is increased by one pressure unit. The specific storage of the element can then be expressed by any of the following equalities

$$s_s' = \alpha\gamma_w = \frac{\gamma_w}{E_c} = \frac{a_v\gamma_w}{1 + e} = \frac{\Delta e\gamma_w}{\Delta\bar{\sigma}(1 + e)} \quad (12)$$

In soil mechanics practice, the expression $a_v\ \gamma_w/(1 + e)$ is the favored term but is seldom utilized. Instead, soils engineers recognize that both compressibility and permeability of a clay layer in the range between its initial and ultimate void ratio are only two of numerous factors that determine the time to attain a certain degree of consolidation. To reduce the number of variables, the coefficient of consolidation (c_v) has been introduced to include combined effects of compressibility and permeability, and its value is determined in the laboratory. The parameter c_v is given as $k'(1 + e)/a_v\gamma_w$. Therefore, in soil mechanics literature, (5) is generally expressed as

$$\partial^2 u/\partial z^2 = (1/c_v)\ (\partial u/\partial t) \quad (13)$$

However, it is clear that c_v is expressed equally well by the product of permeability and the reciprocal of any one of the expressions for specific storage.

Specific storage of natural sediments. Assuming that Hooke's law applies, the specific storage of a perfectly homogeneous confining bed is a constant. As demonstrated in the preceding section, notations available to express this parameter are numerous, depending mainly on the discipline involved. In soil mechanics literature, $a_v\gamma_w/(1 + e)$ is used almost exclusively, In groundwater hydrology, the notation γ_w/E_c is possibly preferable because of use of the bulk modulus of compression in groundwater literature [*Jacob*, 1940]. Variations in the bulk modulus of compression and specific storage based on compressibility alone are shown in Table 1.

Although a wide range for supposedly similar lithologies is indicated, it appears that considerable water is stored in sediments of low permeability that commonly constitute the bulk of aquifer-aquitard systems. Special attention should be given to comparison of values for dense sands, sandy gravels, and clays. The given values for coarse clastics compare favorably with the specific storage of compressible aquifers, a parameter easily calculated from field tests. On the other hand, values for the specific storage of clay beds can be obtained from consolidation tests on undisturbed samples or from a modification of the theory of leaky aquifers developed by *Hantush* [1960]. Because of the time required for pumping to affect a confining layer such that its full thickness contributes to the replenishment of a pumped aquifer, there is likely to be little resemblance between values given in Table 1 and values obtained by Hantush's model.

Recalling that the coefficient of consolidation equals the product of the permeability and the reciprocal of specific storage, the specific storage can be stated as

$$s_s' = k'/c_v \qquad (14)$$

This expression is particularly useful because of the fund of laboratory determinations of the parameters involved. For example, data reported from results of an extensive drilling and testing program carried out by the U. S. Geological Survey in the Los Banos-Kettleman City area, California [*Johnson and Morris*, 1962], are suited to this analysis. Undisturbed samples from test holes drilled to depths of 1000 to 2200 feet were tested by the Survey for coefficients of consolidation and permeability and classified according to the Unified Soil Classification System. When these data are used in (14), calculations of specific storage are between 1 and 2 orders of magnitude smaller than those reported in Table 1. The discrepancy occurs because permeability values calculated from consolidation-test data are small owing to large porosity decline under loads required to consolidate the samples. As pointed out by Johnson and Morris, permeability determinations are most meaningful if they are computed at the void ratio existing under effective overburden pressure.

In the same study, undisturbed samples immediately adjacent to samples subjected to laboratory consolidation were tested by the Survey in a variable-head permeameter under no load. These data, when used in (14) with consolidation-test data, provide a range in specific storage of the order of magnitude reported in Table 1. For six samples classified as CL (inorganic clays of low to medium plasticity), the specific storage ranged from 3.3×10^{-4} to 1.9×10^{-3} and averaged 8.9×10^{-4}. For nine samples classified as CH (inorganic clays of high plasticity), the specific storage ranged from 1.4×10^{-4} to 2.7×10^{-2} and averaged 4.6×10^{-3}. Based on average values of specific storage, the bulk modulus of compression for CL and CH material is calculated to be 7.0×10^4 and 1.4×10^4, respectively.

TABLE 1. Range in Values for the Bulk Modulus of Compression (E) and Specific Storage (γ_w/E) (Modified after *Jumikis*, 1962)

Material	E (lb/ft^2)	γ_w/E (lb/ft)
Plastic clay	1×10^4 to 8×10^4	6.2×10^{-3} to 7.8×10^{-4}
Stiff clay	8×10^4 to 1.6×10^5	7.8×10^{-4} to 3.9×10^{-4}
Medium hard clay	1.6×10^5 to 3×10^5	3.9×10^{-4} to 2.8×10^{-4}
Loose sand	2×10^5 to 4×10^5	3.1×10^{-4} to 1.5×10^{-4}
Dense sand	1×10^6 to 1.6×10^6	6.2×10^{-5} to 3.9×10^{-5}
Dense sandy gravel	2×10^6 to 4×10^6	3.1×10^{-5} to 1.5×10^{-5}
Rock, fissured, jointed	3×10^6 to 6.25×10^7	2.1×10^{-5} to 1×10^{-6}
Rock, sound	Greater than 6.25×10^7	Less than 1×10^{-6}